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FRICTION OF SOLID BODIES WITH FORMATION OF A MELT LAYER

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Some laws governing friction of solid bodies with formation of a developed layer of melt are studied. In contrast to [1], we study a case when a finite body moves along a melting substrate (half-plane) with the heat required for melting being liberated by frictional heat release or from a heat flux from the body. To determine the outflow of heat from the melt layer to the half-plane and the zone of melt in front of the moving body we considered two plane boundary-value problems of the equation of heat conduction.

1. Laws Governing Melt-Film Formation in High-Speed Friction. In friction of solid bodies with a high relative velocity and at an elevated temperature, transition of the material of the rubbing bodies to a plastic or liquid state is possible in the zone of frictional contact [1-3]. In what follows we consider some laws governing the friction of bodies for a developed melt layer when the relative velocity of motion or temperature of one of the bodies is rather high and the thickness of the melt layer is $\delta > 10^{-5}-10^{-4}$ cm.

A plane model problem of the formation of a melt film in friction of smooth bodies (the thickness of the film is much larger than the characteristic roughness) is considered. To describe the motion of the melt we use the equations of a viscous fluid. It is assumed that a sufficient thickness of the film allows one to neglect plastic deformations in rubbing surfaces and physical effects of the type of disjoining pressure in thin films.

In friction of a body of limited dimensions on a substrate, melting of both the moving body and the substrate is possible, depending on the parameters of the problem. In the first case, consumption (rubbing off or wear) of the substance of the body and, consequently, the presence of a component of the body velocity toward the contact zone have a substantial effect. The dependence of the rate of wear (rate of melting) on the determining parameters along with the dependence of the force of friction are determined in solution of the problem. The load – the force pressing the body against the substrate – is assumed to be known. As is shown in [1, 4], the assumption of a melt layer of constant thickness is justified, which makes it possible to obtain dependences for the rate of melting and the force of friction from the balance relations of mass and heat in the melt film. A drawback of the assumption is a reduction of the order of the determining system of equations and the inevitable relationship between the pressing force and the consumption of melt in the initial cross section. In [1] the natural assumption that consumption in the initial cross section equals zero and the entire melt is entrained by a moving plane is adopted. In spite of the specific value of load corresponding to this solution Y_0, it can be used in virtually all applications, since the correction to it allowing for the actual load value Y is on the order of

$$\frac{\delta^2}{2\mu}\frac{dp}{Udx} \sim \frac{\delta^2}{l^2}\frac{Y - Y_0}{2\mu U} << 1$$

and, as a rule, is small due to the small thickness of the melt layer.

In [4] a case of melting of a body on a slowly moving heated substrate in which with a sufficient value of pressing force the reduced parameter is higher than unity and, depending on the value of the load, a portion of the melt is pressed out in front of the body is considered. A more accurate formulation, which is based on the method of integral relations of boundary-layer theory and leads to the necessity of integration of a system of ordinary differential equations, is developed in [5].

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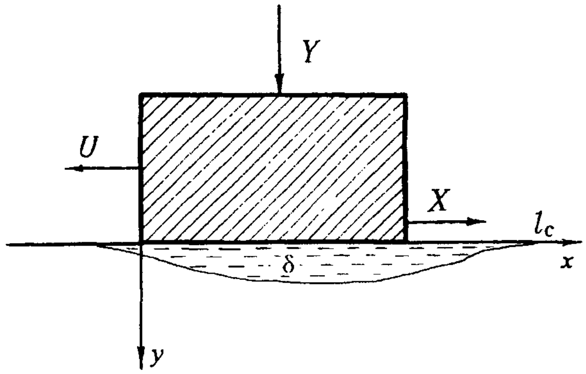

Fig. 1. Simplified scheme of frictional contact in friction of heated or thermally insulated body on melting substrate.

If in high-speed friction of solid bodies the substrate is melted, then account for the variable thickness of a melt layer is of principal importance. In [6] the problem of substrate melting in motion of a heated body is considered by an asymptotic method in a self-similar formulation. An alternative approach is based on exact solution of the system of equations, in which small terms are discarded. Estimates show that the flow of liquid in the melt film can be described in an approximation of lubrication. In fact, the convective terms in the equation of inflow of heat are on the order of EcPr, if the substrate melts due to viscous dissipation in the contact zone, or on the order of Ste, if the substrate melts in contact with a moving heated body [7]. The convective terms in the equation of momentum are on the order of Ec or Ste/Pr. Thus, a lubrication approximation can be used if Ec, Ste/Pr << 1 (for a fast-moving thermally insulated body) or Ste, Ste/Pr << 1 (for a heated body). The system of equations in a lubrication approximation that describes the flow in the melt film has the form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\,, \quad k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 = 0\,, \quad -\frac{dp}{dx} + \mu\frac{\partial^2 u}{\partial y^2} = 0\,. \tag{1.1}$$

Here x, y are Cartesian coordinates associated with the moving body; the axis y is directed toward the melt layer (Fig. 1).

2. Plane Boundary-Value Problems of Heat Conduction. In general, the study of problems of heat and mass transfer with phase transitions is complicated by the necessity of simultaneous solution of the hydrodynamic and thermal parts of the problem in the melt region and the thermal problem in a solid phase with an unknown boundary – the front of the phase transition – the position of which is determined by an additional boundary condition – the Stefan condition. However, in a number of formulations, a solution of the thermal problem in a solid phase and, consequently, the heat flux from the melt zone can be found irrespective of the shape of the melt layer. Often the hydrodynamic part of the problem can be considered separately after solution of the combined thermal problem in the melt layer and solid and determination of the position of the front.

As an example of determination of heat outflow to the solid phase, we consider the familiar one-dimensional solution of the classical Stefan problem for the equation of heat conduction. Let the melt front $x = 0$ move at a constant velocity $-U$ inside a solid substance $\{x < 0\}$ with temperature T_0 [8]:

$$\rho_s c_s U\frac{dT}{dx} = k_s\frac{d^2 T}{dx^2}\,, \quad x \le 0\,;$$

$$x = 0:\ \ T = T_m\,, \quad \left[k\frac{dT}{dx}\right]_{-0}^{+0} = \rho_s UL\,, \quad x = -\infty:\ \ T = T_0\,. \tag{2.1}$$

Using two boundary conditions, we can determine the profile of the temperature and heat outflow to the solid substance. Then the Stefan condition can be written in the form

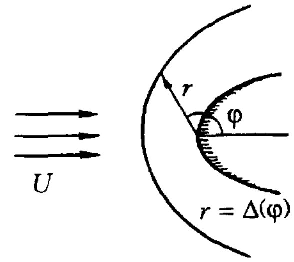

Fig. 2. Plane Stefan problem: formation of melt layer in melting of half-plane by heated body.

$$k \left. \frac{dT}{dx} \right|_{+0} = \rho_s UL + \rho_s c_s U (T_m - T_0) = \rho_s UL_e .$$

Thus, for the one-dimensional formulation, it follows from the assumption of stationarity of the temperature profile in the solid substance in the system of coordinates associated with the moving melt front that allowance for heat removal to the solid substance is equivalent to use of the effective quantity $L_e = L + c_s(T_m - T_0)$ instead of the specific heat of melting.

Within the framework of the same one-dimensional formulation we assume that the solid substance melts in motion of a heat source $\{x \geq l_T\}$ with temperature $T_w > T_m$. When EcPr << Ste << 1, a melt layer is formed as a result of heat transfer, and the convective terms and viscous dissipation in the equation of heat inflow can be neglected. Thus, the thermal problem is separated from the hydrodynamic and for the melt layer we have the following boundary-value problem:

$$k \frac{d^2T}{dx^2} = 0 ,$$

$$x = l_T : \quad T = T_w , \quad x = 0 : \quad T = T_m , \quad k \left. \frac{dT}{dx} \right|_{+0} = \rho_s UL_e .$$

Using three boundary conditions, we find the thickness of the melt layer:

$$l_T = \frac{\nu}{U} \frac{\mathrm{Ste_e}}{\mathrm{Pr}} \frac{1}{N} , \quad N = \frac{\rho_s}{\rho} , \quad \mathrm{Ste_e} = \frac{c_v (T_w - T_m)}{L_e} . \tag{2.2}$$

The one-dimensional problem is physically inconsistent, since it is necessary to organize the removal of the melt in a special manner (e.g., absorption of it by the surface of a heat source moving at velocity NU). However, this formulation can be used in melting of a solid substance by a blunt heat source. If EcPr << Ste << 1 and the curvature radius of the heat source at a front point $R >> l_T$, then the formulated relation is applicable near it and the thickness of the melt layer is prescribed by formula (2.2). Since in this case the Reynolds number over the size of the melt zone is small, $\mathrm{Ste_e}/N\mathrm{Pr}$, the fields of velocity and pressure can be determined from the solution of the equations in the Stokes approximation.

We now consider a plane formulation of the Stefan problem. In a polar system of coordinates $\{r, \varphi\}$ with a pole at the critical point of the moving heat source (Fig. 2) the Stefan condition has the form:

$$\rho_s UL (\sin \varphi \Delta' + \cos \varphi \Delta) = \left[k \left(\frac{\partial T}{\partial r} \Delta - \frac{\Delta'}{\Delta} \frac{\partial T}{\partial \varphi} \right) \right]_{r=\Delta+0}^{r=\Delta-0} . \tag{2.3}$$

Here $r = \Delta(\varphi)$ is the equation of the melting front. An obvious solution of the plane problem is an analog of the one-dimensional solution presented above. We consider this case of a principally different geometry of the heat source by the example of a beam $\{\varphi = 0\}$ moving in a longitudinal direction. At EcPr << Ste << 1, when the temperature satisfies the Laplace equation in the melt layer and the heat outflow to the solid substance is neglected (we let $T_0 = T_m$ or take into account the outflow in a model manner by substituting L for L_e), it is not difficult to find a solution of the boundary-value problem

$$\Delta T_1 = 0\,,\ \ 0 \le r_1 \le \Delta_1\,,\ \ 0 \le \varphi \le 2\pi\,,$$

$$T_1\,(r_1, 0) = T_1\,(r_1, 2\pi) = 1\,,$$

$$T_1\,(0, \varphi) = 1\,,\ \ T_1\,(\Delta_1, \varphi) = 0\,, \tag{2.4}$$

$$\sin\varphi\Delta_1' + \cos\varphi\Delta_1 = \frac{\partial T_1}{\partial r_1}\Delta_1 - \frac{\Delta_1'}{\Delta_1}\frac{\partial T_1}{\partial\varphi}\,,\ \ r_1 = \Delta_1\,(\varphi)\,,$$

$$\Delta_1\,(\varphi) = \frac{1}{2\sin^2\frac{\varphi}{2}}\,,\ \ T_1\,(r_1, \varphi) = 1 - \sqrt{2r_1}\,\sin\frac{\varphi}{2}\,.$$

Here $r_1 = r/l_T$, $T_1 = (T - T_m)/(T_w - T_m)$ are dimensionless variables. The melt front has a parabolic shape. With distance from the edge of the beam, the temperature profile changes from $T_1(r_1) = 1 - \sqrt{2r_1}$ at $\varphi = \pi$ to linear along the coordinate across the beam. The heat flux has a singularity $r^{-1/2}$ at $r = 0$. Since $\Delta(\pi) = l_T/2$, other parameters being equal, the thickness of the melt layer in front of the plate is half that in front of the body, which has a front point. The constructed solution is valid if the thickness of some heat source on the scale $l > l_T$ can be neglected.

In friction of a solid body on a melting substrate, allowance for the heat outflow from the melt zone to the surrounding solid substance is important. If the melt layer is rather thin, then the heat outflow can be determined assuming that a heat spot with temperature T_m moves along the substrate surface. This scheme is applicable if the thickness of the temperature boundary layer $l/\sqrt{\mathrm{Pe}}$ is much larger than the thickness of the melt layer. For a thermally insulated body it is necessary that $\sqrt{\mathrm{EcPr}} \ll 1$ [7]. If the substrate melts in contact with a moving heated body, then the scheme with a heat spot is applicable when $\sqrt{\mathrm{Ste}} \ll 1$. The dependence of heat outflow on the distance x from the spot edge can be determined from the self-similar solution for a temperature boundary layer

$$q\,(x) = \sqrt{\left(\frac{\rho_s c_s k_s U}{\pi x}\right)}\,(T_m - T_0)\,,$$

$$\frac{q\,(x)}{\rho_s U L} = \frac{1}{\sqrt{\pi}}\sqrt{\left(\frac{l}{x}\right)}\,\frac{\mathrm{Ste_w}}{\sqrt{\mathrm{Re\,Pr}}}\sqrt{\left(\frac{k_s c_s}{k c_v N}\right)}\,,\ \ \mathrm{Ste_w} = \frac{c_v\,(T_m - T_0)}{L}\,. \tag{2.5}$$

3. Melting of Substrate Due to Frictional Heat Release in Contact with a Moving Thermally Insulated Body. A case of high-speed melting of solid substance by a thermally insulated thin body is studied in [7] in the same approximation. By symmetry, the problem corresponds to a partial case of melting of substance by a moving plate (the surface of the substrate beyond the zone of frictional contact is assumed to be thermally insulated). In this case the dependences are simplified and can be presented in a finite form.

The boundary conditions for system of equations (1.1) are

$$y = 0:\ \ \frac{\partial T}{\partial y} = 0\,,\ \ u = 0\,;$$

$$y = \delta:\ \ \rho_s U L \delta'\,(x) = \left[-\,k\frac{\partial T}{\partial y}\right]_{y=\delta+0}^{y=\delta-0}\,,\ \ u = U\,,\ \ v = (1 - N)\,U\delta'\,(x)\,,\ \ T = T_m\,. \tag{3.1}$$

The quadratic velocity profile is determined from the equation of momentum and the boundary conditions. The pressure gradient in the layer is found by the first equation of (1.1), which is integrated with respect to the layer thickness (balance of consumption). In finding the integration constant we use an initial condition $\delta(0) = 0$ that is natural for a thermally insulated body. Substitution of the velocity profile into the equation of heat inflow gives an ordinary differential equation for $\delta(x)$

$$\delta' + \frac{q(x)}{\rho_s UL} = \frac{1 + 3(2N-1)^2}{\rho_s L}\frac{\mu U}{\delta}. \tag{3.2}$$

Using formula (2.3) for heat outflow $q(x)$, we have

$$\delta' + \frac{1}{\sqrt{\pi}}\sqrt{\left(\frac{l}{x}\right)}\frac{\mathrm{Ste_w}}{\sqrt{\mathrm{Re\,Pr}}}\sqrt{\left(\frac{k_s c_s}{k c_v N}\right)} = \frac{1 + 3(2N-1)^2}{N}\frac{\mathrm{Ec}}{\mathrm{Re}}\frac{l}{\delta}. \tag{3.3}$$

The solution of Eq. (3.3) in dimensionless variables is

$$\delta_1 = \frac{\beta\delta}{l},\quad x_1 = \frac{x}{l},\quad \beta = \sqrt{\left(\frac{\mathrm{Re}}{\mathrm{Ec}}\right)},\quad \delta_1 = A\sqrt{x_1},$$

$$A = \sqrt{\left(\frac{2[1 + 3(2N-1)^2]}{N} + \frac{\mathrm{Ste_w^2}}{\pi\,\mathrm{Ec\,Pr}}\frac{k_s c_s}{k c_v N}\right)} - \frac{\mathrm{Ste_w}}{\sqrt{\pi\,\mathrm{Ec\,Pr}}}\sqrt{\left(\frac{k_s c_s}{k c_v N}\right)}.$$

For weak and strong heat removal we obtain

$$A = \sqrt{\left(\frac{2[1 + 3(2N-1)^2]}{N}\right)} - \frac{\mathrm{Ste_w}}{\sqrt{\pi\,\mathrm{Ec\,Pr}}}\sqrt{\left(\frac{k_s c_s}{k c_v N}\right)} \qquad (\mathrm{Ste_w} \ll \sqrt{\mathrm{Ec\,Pr}}),$$

$$A = \frac{1 + 3(2N-1)^2}{N}\frac{\sqrt{\pi\,\mathrm{Ec\,Pr}}}{\mathrm{Ste_w}}\sqrt{\left(\frac{k c_v N}{k_s c_s}\right)} \qquad (\mathrm{Ste_w} \gg \sqrt{\mathrm{Ec\,Pr}}).$$

If we neglect the variation of the thermophysical characteristics of the medium in phase transition, then,

$$A = \sqrt{8} - \frac{\mathrm{Ste_w}}{\sqrt{\pi\,\mathrm{Ec\,Pr}}},\quad A = \frac{4\sqrt{\pi\,\mathrm{Ec\,Pr}}}{\mathrm{Ste_w}}.$$

The dependence of the pressure in the layer on the longitudinal coordinate has a form

$$p(x) = -\frac{6\mu U(2N-1)\beta^2}{A^2 l}\ln\left(\frac{x}{l}\right) = -\frac{6(2N-1)}{A^2}\rho L\ln\left(\frac{x}{l}\right). \tag{3.4}$$

Here we took into account that the pressure in the last cross section of the melt layer was constant $p(l) = 0$. For the force of friction

$$X = \int_0^l \mu\left.\frac{\partial u}{\partial y}\right|_{y=0} dx = 4\mu U(3N-1)\frac{\beta}{A} = \frac{4\mu U(3N-1)}{A}\sqrt{\left(\frac{\mathrm{Re}}{\mathrm{Ec}}\right)}. \tag{3.5}$$

If we neglect the variation of the thermophysical characteristics of the medium, then

$$X = 2\sqrt{2}\,\mu U\sqrt{\left(\frac{\mathrm{Re}}{\mathrm{Ec}}\right)}\left(1 + \frac{\mathrm{Ste_w}}{\sqrt{8\pi\,\mathrm{Ec\,Pr}}}\right) \sim U^{1/2} \qquad (\mathrm{Ste_w} \ll \sqrt{\mathrm{Ec\,Pr}}),$$

$$X = 2\mu U \sqrt{\left(\frac{\mathrm{Re}}{\mathrm{Ec}}\right)} \frac{\mathrm{Ste_w}}{\sqrt{\pi\, \mathrm{Ec\, Pr}}} \sim U^{-1/2} \quad (\mathrm{Ste_w} >> \sqrt{\mathrm{Ec\, Pr}}\,).$$

The load value is determined by the formula

$$Y = \int_0^l p dx = \frac{6\,(2N-1)}{A^2} \rho L l . \tag{3.6}$$

At smaller values of the pressing force, a solution of the problem in this formulation does not exist (a melt layer can originate inside the zone of frictional contact). The assumption that $\delta(0) = 0$ (contact between body and substrate) leads to the fact that at larger values of the pressing force a portion of load is compensated by a concentrated force applied to the body in the cross section $x = 0$.

4. Melting of Substrate in Contact with a Moving Heated Body. As is shown above (Sec. 2), when $T_w > T_m$, a melt region of a thickness $l_T/2$ is formed in front of the heated body. The contribution of this zone to the balance of mass and heat in the melt layer $\sim l_T/l = \mathrm{Ste/PrRe}$ is negligibly small, since in the approximation considered the terms $\sim 1/\mathrm{Re}$ are neglected. Within the framework of the equations of lubrication, we have for $\delta(x)$

$$\delta' + \frac{q\,(x)}{\rho_s UL} = \frac{3 - 8N + 6N^2}{\rho_s L} \frac{\mu U}{\delta} + \frac{k\,(T_w - T_m)}{\rho_s UL\delta} .$$

Using formula (2.5) for the heat outflow $q(x)$, we obtain

$$\delta' + \frac{1}{\sqrt{\pi}} \sqrt{\left(\frac{l}{x}\right)} \frac{\mathrm{Ste_w}}{\sqrt{\mathrm{Re\, Pr}}} \sqrt{\left(\frac{k_s c_s}{k c_v N}\right)} = \left(\frac{3 - 8N + 6N^2}{N} \frac{\mathrm{Ec}}{\mathrm{Re}} + \frac{\mathrm{Ste}}{N\,\mathrm{Re\, Pr}}\right) \frac{l}{\delta} . \tag{4.1}$$

The solution of Eq. (4.1) in dimensionless variables is

$$\delta_1 = \frac{\beta\delta}{l}, \quad x_1 = \frac{x}{l}, \quad \beta = \sqrt{\left(\frac{\mathrm{Re\, Pr}}{\mathrm{Ste}}\right)}, \quad \delta_1 = A \sqrt{x_1},$$

$$A = \sqrt{\left(\frac{2}{N}\left(1 + (6N^2 - 8N + 3)\frac{\mathrm{Ec\, Pr}}{\mathrm{Ste}}\right) + \frac{\mathrm{Ste_w^2}}{\pi\, \mathrm{Ste}} \frac{k_s c_s}{k c_v N}\right)} - \frac{\mathrm{Ste_w}}{\sqrt{\pi}\, \mathrm{Ste}} \sqrt{\left(\frac{k_s c_s}{k c_v N}\right)} .$$

For weak and strong heat removal

$$A = \sqrt{\left(\frac{2}{N}\left(1 + (6N^2 - 8N + 3)\frac{\mathrm{Ec\, Pr}}{\mathrm{Ste}}\right)\right)} - \frac{\mathrm{Ste_w}}{\sqrt{\pi}\, \mathrm{Ste}} \sqrt{\left(\frac{k_s c_s}{k c_v N}\right)},$$

$$\mathrm{Ste_w^2} << \mathrm{Ste} + (6N^2 - 8N + 3)\,\mathrm{Ec\, Pr};$$

$$A = \frac{\mathrm{Ste} + (6N^2 - 8N + 3)\,\mathrm{Ec\, Pr}}{N\sqrt{\mathrm{Ste}}\,\mathrm{Ste_w}} \sqrt{\left(\frac{\pi k c_v N}{k_s c_s}\right)},$$

$$\mathrm{Ste_w^2} >> \mathrm{Ste} + (6N^2 - 8N + 3)\,\mathrm{Ec\, Pr}.$$

If we neglect variations of the thermophysical characteristics of the medium, then

$$A = \sqrt{\left(2\left(1 + \frac{\mathrm{Ec\, Pr}}{\mathrm{Ste}}\right)\right)} - \frac{\mathrm{Ste_w}}{\sqrt{\pi}\, \mathrm{Ste}}, \quad A = \frac{\sqrt{\pi}\,(\mathrm{Ste} + \mathrm{Ec\, Pr})}{\sqrt{\mathrm{Ste}}\,\mathrm{Ste_w}} .$$

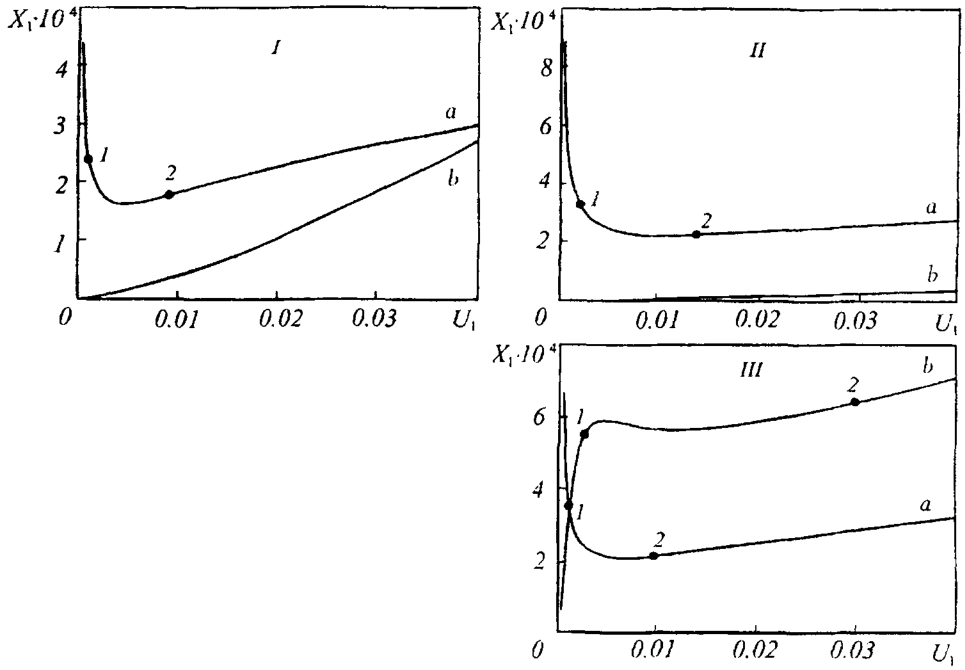

Fig. 3. Dimensionless force of friction X_1 as a function of dimensionless relative velocity of motion U_1 (the substrate material and dimensionless parameters of friction are given in Table 1). Curves *a* and *b* correspond to friction of thermally insulated and heated bodies.

TABLE 1.

Position in the figures	Material	Pr	Re₀	Ste	Ste$_w$
I	Ice	11.4	$3.85 \cdot 10^6$	0.07	0.040
II	Aluminum	0.028	$5.33 \cdot 10^6$	0.01	0.005
II	Ice	11.4	$3.85 \cdot 10^6$	$5 \cdot 10^{-5}$	0.064

The dependence of pressure in the layer on the longitudinal coordinate, load, and friction force are determined by formulas similar to (3.4)-(3.6):

$$p(x) = -\frac{6\mu U(2N-1)\beta^2}{A^2 l} \ln\left(\frac{x}{l}\right) = -\frac{6\mu U(2N-1)}{A^2 l}\frac{\mathrm{Re\,Pr}}{\mathrm{Ste}} \ln\left(\frac{x}{l}\right),$$

$$Y = \frac{6\mu U(2N-1)}{A^2}\frac{\mathrm{Re\,Pr}}{\mathrm{Ste}}.$$

At smaller load values, a solution of the problem in this formulation does not exist. At higher values of the pressing force, melting inside the substrate begins. In this case

$$X = \frac{4\mu U(3N-1)}{A}\sqrt{\left(\frac{\mathrm{Re\,Pr}}{\mathrm{Ste}}\right)}.$$

In the case of weak heat removal

$$X \sim U^{3/2}\ (\mathrm{Ste} \gg \mathrm{Ec\,Pr}),\quad X \sim U^{1/2}\ (\mathrm{Ste} \ll \mathrm{Ec\,Pr}),$$

with strong heat removal

$$X \sim U^{3/2}\ (\mathrm{Ste} \gg \mathrm{Ec\,Pr})\,, \quad X \sim U^{-1/2}\ (\mathrm{Ste} \ll \mathrm{Ec\,Pr})\,.$$

As an example of use of the obtained relations we consider the behavior of the force of friction as a function of the velocity of motion under real conditions of high-speed friction. The different character of the dependence $X(U)$, which is determined by thermal conditions in the contact zone, is of interest. Figure 3 presents the curves $X_1(U_1)$, where $X_1 = X/\rho Ll$ is the dimensionless force of friction, $U_1 = \sqrt{\mathrm{Ec}} = U/\sqrt{L}$ is the dimensional velocity of motion. Variation of thermophysical parameters of the substrate material in phase transition is neglected. Since the Reynolds number changes with the velocity of sliding, the quantity $\mathrm{Re}_0 = \mathrm{Re}/\sqrt{\mathrm{Ec}} = l\sqrt{L}/\nu$ is assumed to be constant.

Curves a and b correspond to the friction of a thermally insulated and a heated body, respectively. The thickness of the melt layer is $\delta = 10^{-5}$ and 10^{-4} cm ($l = 1$ cm) at points 1 and 2, and in the range of parameters between them a transition to the mode of friction with a developed melt layer occurs. The curve $X_1(U_1)$ for a heated body passes through the coordinate origin, since even at a small velocity of motion a sufficient thickness of the melt layer is provided by the heat flux from the body. For a thermally insulated body, the initial portion of the curve is, as a rule, characterized by reduction of the force of friction due to a sharp increase in the thickness of the melt layer. This probably explains the experimentally observed substantial decrease in the force of friction with an increase in the velocity of motion and formation of the melt layer in the zone of frictional contact. At a large velocity of motion, the weak heat removal is asymptotic and the force of friction increases with the velocity. At rather large velocity of motion, the force of friction for a heated body becomes equal to that for a thermally insulated body (Fig. 3, *I*). The intersection of curves b and a corresponds to the instant when the maximum temperature in the melt layer exceeds the temperature of the body, the thickness of the layer is restrained by the heat flux inside the moving heated body, and the force of friction is, therefore, higher than in the case of a thermally insulated body. In Fig. 3, *II* the point of intersection shifts to the region of high velocities of motion, since the Prandtl number is small for aluminium (liquid metals) and the effect of viscous dissipation on thermal processes in the layer is weaker. Figure 3, *III* presents the situation of strong heat removal $\mathrm{Ste}_w^2 \gg \mathrm{Ste}$, when for a heated body the curve $X_1(U_1)$ has a valley in the range of the velocity of motion $\mathrm{Ste} \ll U_1^2\mathrm{Pr} \ll \mathrm{Ste}_w^2$.

5. Length of Melt Zone Behind Body. A solution of the problem of collapse of a cavity behind a moving heat source of an arbitrary nature in approximation of a thin layer is constructed in [7]. The flow in the wake region is characterized by a constant velocity profile $u(x, y) \equiv UN$ and temperature $T(x, y) \equiv T_m$, and the length of the melt cavity is determined by the relation

$$\rho_s UL\delta\,(l) = \int_l^{l+l_c} q\,(x)\,dx\,. \tag{5.1}$$

Here $\delta(l)$ is the thickness of the melt layer in the cross section $x = l$. The formula gives the lower bound for the length of the melt layer, since the temperature in the cross section $x = l$ is $T(l, y) \geq T_m$. Estimate (5.1) is the more accurate, the smaller the corresponding Stefan number $\mathrm{Ste}_m = c_v(T_{max} - T_m)/L$, $T_{max} = \max\limits_{y \in [0,\,\delta(l)]} T(l, y)$. For a thermally insulated body $\mathrm{Ste}_m \sim \mathrm{EcPr} \ll 1$, with account for heat inflow from the body $\mathrm{Ste}_m \sim \mathrm{Ste} + \mathrm{EcPr} \ll 1$. Using formula (2.5) for the heat outflow $q(x)$ to the solid substance, we obtain

$$\sqrt{\left(1 + \frac{l_c}{l}\right)} = 1 + \frac{\delta\,(l)}{2l}\,\frac{\sqrt{\pi\,\mathrm{Re\,Pr}}}{\mathrm{Ste}_w}\,\sqrt{\left(\frac{kc_vN}{k_s c_s}\right)}\,. \tag{5.2}$$

For a thermally insulated body

$$\sqrt{\left(1 + \frac{l_c}{l}\right)} = 1 + \frac{A}{2}\,\frac{\sqrt{\pi\,\mathrm{Ec\,Pr}}}{\mathrm{Ste}_w}\,\sqrt{\left(\frac{kc_vN}{k_s c_s}\right)}\,.$$

With weak heat removal $l_c/l = B/2 >> 1$, and with strong heat removal $l_c/l = B << 1$,

$$B = [1 + 3\,(2N - 1)^2]\,\frac{\pi\,\mathrm{Ec}\,\mathrm{Pr}}{\mathrm{Ste}_w^2}\,\frac{kc_v}{k_s c_s}.$$

In a case of a heated body the relations are

$$\sqrt{\left(1 + \frac{l_c}{l}\right)} = 1 + \frac{A}{2}\,\frac{\sqrt{\pi\,\mathrm{Ste}}}{\mathrm{Ste}_w}\,\sqrt{\left(\frac{kc_v N}{k_s c_s}\right)},$$

$$\frac{l_c}{l} = \frac{B}{2}\quad\left(\frac{l_c}{l} >> 1\right),\quad \frac{l_c}{l} = B\quad\left(\frac{l_c}{l} << 1\right),\quad B = \frac{\pi\,[\mathrm{Ste} + (6N^2 - 8N + 3)\,\mathrm{Ec}\,\mathrm{Pr}]}{\mathrm{Ste}_w^2}\,\frac{kc_v}{k_s c_s}.$$

NOTATION

U, velocity of motion of body relative to substrate; Y, load (force pressing body against substrate); X, force of friction between body and substrate; l, length of body (zone of frictional contact); l_T, length of melt zone in front of body, $l_T = \frac{\nu}{U}\frac{\mathrm{Ste_e}}{\mathrm{Pr}}\frac{1}{N}$; l_c, length of wake (cavity) of melt behind body; T_w, temperature of body; T_m, melting point of substrate material; T_0, temperature of substrate material at a distance from melt zone; δ, thickness of melt layer; p, pressure in melt layer; μ, dynamic viscosity; ρ, density; $\nu = \mu/\rho$, kinematic viscosity; c_v, heat capacity; k, thermal conductivity of melt; $\lambda = k_s/\rho_s c_s$, thermal diffusivity of substrate material in solid phase; q, heat outflow from melt zone to solid phase; L, specific heat of melting; $L_e = L + c_s(T_m - T_0)$, effective specific heat of melting of substrate material; $\mathrm{Pr} = c_v\mu/k$, Prandtl number of melt; $\mathrm{Re} = Ul/\nu$, Reynolds number for melt layer; $\mathrm{Ec} = U^2/L$, Eckert number; $\mathrm{Ste} = c_v(T_w - T_m)/L$, Stefan number; $\mathrm{Ste_e} = c_v(T_w - T_m)/L_e$, Stefan number for effective specific heat of melting; $\mathrm{Ste_w} = c_v(T_m - T_0)/L_e$, Stefan number characterizing heat removal from melt zone to substrate with temperature T_0 at distance from zone of contact; $N = \frac{\rho_s}{\rho}$, ratio of densities of solid and liquid phases for substrate material; $\mathrm{Pe} = Ul/\lambda$, Peclet number; Re_0, modified Reynolds number. Subscripts: T, related to heated body; c, related to cavity of the melt; w, related to wall; m, related to melting; 0, characterizes an "undisturbed" state; s, related to solid phase; e, at "effective" parameters; 1, at dimensionless parameters not having a simple physical interpretation.

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